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Schrödinger Approach to Ground State Wavefunctionals

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Abstract

The nonperturbative structure of the QCD vacuum is studies in two and four dimensions using a Schrödinger approach to quantum field theory.

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Introduction.

The Schrödinger approach to quantum field theory permits a direct study of the vacuum structure through the analysis of the vacuum wavefunctionals of the theory. Various properties of this vacuum structure should reflect the nature of chiral symmetry breaking, color confinement and other nonperturbative features of the complete theory. The Schrödinger approach may also allow the study of vacuum structure through the application of variational methods and the use of trial wavefunctionals.

In this talk, I will present some results of using the Schrödinger approach to study quantum chromodynamics in two and four dimensions. In two dimensions, QCD can be systematically analysed[1] through the use of the large N_C expansion where N_C is the number of colors. In this case, we can study both the vacuum structure and the nature of the elementary excitations of the system. In four dimensions, nonperturbative aspects of the vacuum structure of both the gluon and quark wavefunctionals have implications for color confinement and chiral symmetry breaking.

QCD in Two Dimensions.

Quantum chromodynamics describes the interactions of colored quarks with the color Yang-Mills gauge fields. In two dimensions there are no transverse gauge degrees of freedom in the gauge fields, and the gauge fields may all be eliminated by an appropriate choice of gauge. Indeed, 't Hooft[2] used the light-cone gauge to study the spectrum of the meson bound states. Much further work was also done within this framework[3], on the structure of the scattering amplitudes and many other properties of the theory. Here, confinement is not the issue as the light-cone gauge produces a confining linear potential in two dimensions. However, the light-cone gauge can not easily be used to study questions related to the vacuum structure. These questions can be analysed by combining the usual coulomb gauge formulation with the large N_C limit.

The large N_C limit is defined by the formal limit where the gauge coupling constant, $\alpha_C = g^2{}_C/4\pi$, is taken to zero with the combination, $\alpha_C \cdot N_C$ fixed. In this limit, the nonabelian gauge structure dictates that the leading contributions are given by planar diagrams with the fewest number of internal quark loops. Nonplanar gluon interactions are suppressed by $O(1/N_C^2)$ and an internal quark loops by $O(1/N_C)$.

The standard coulomb gauge is specifed by $\nabla\cdot\mathbf{A} = 0$ which implies $\mathbf{A} = 0$ in two dimensions. Hence, the gauge fields generate only the usual coulomb potential interaction between the quark fields. These interactions are fully contained in the coulomb gauge Hamiltonian

$$\mathbf{H} = \int dx\, \Psi^{a+}(\alpha\cdot\nabla+\beta m)\Psi_a + (g^2{}_C/2)\int dx \int dy\, \mathbf{J}_0{}^a(x)\cdot\mathbf{J}_0{}^a(x)\,(-\nabla^2)^{-1}(x,y) \quad [1]$$

where the color charge density is given by $\mathbf{J}_0{}^a(x) = (1/4)[\Psi^+,\lambda^a\Psi]$.

In leading N_C, this Hamiltonian is of order N_C, and the Hartree approximation is exact. I will use the best plane-wave ground state wavefunctional as a trial vacuum state. Now I expand the quark field in terms of particle and antiparticle creation and destruction operators using a general plane-wave basis

$$\Psi_a(x) = \int dp\,\{u_p f_p(x)\mathbf{A}_{pa} + v_p f^*{}_p(x)\mathbf{B}^+{}_{pa}\} \quad [2]$$

where $f_p(x) = e^{ip\cdot x}/(2\pi)^{1/2}$ and the two component spinors are given by $u_p = (\cos(\theta_p/2), \sin(\theta_p/2))$ and $v_p = (\sin(\theta_p/2, \cos(\theta_p/2))$. This expansion is parameterized by the chiral angles, θ_p, with $\theta_{-p} = -\theta_p$. The trial vacuum state, $|V\rangle_{\theta p}$, is defined as the state annihilated by the particle and antiparticle destruction operators and is a functional of the set of chiral angles, $\{\theta_p\}$. This set of chiral angles can be viewed as variational parameters for the ground state wavefunctional.

The vacuum energy density is simply computed through normal ordering the Hamiltonian using this basis with the result

$$E_0 = \langle V|\mathbf{H}|V\rangle_{\theta p} = (N_C/2\pi)\{ -\int dp\,[p\cdot\sin(\theta_p) + m\cdot\cos(\theta_p)] + \alpha_C\cdot N_C\cdot(1/2)\int dp\int dq\,[\sin^2((\theta_p-\theta_q)/2)/(p-q)^2]\}. \quad [3]$$

It is clear there is no explicit coulomb singularity at the point p=q. The vacuum state is found by varying the chiral angle, θ_p, in Eq.[3]. There is a nontrivial solution for θ_p even in the chiral limit, m=0. In this case the solution has the form given in Fig.[1] where it is compared to the naive massless limit and to the normal massive case. It is clear that an infrared mass is spontaneously generated for small p, but that this mass vanishes quickly at high momenta, consistent with the operator product expansions applied to this system. This mass generation signifies the spontaneous breaking of chiral symmetry in the large N_C limit which can also be seen through the chiral condensate of the mass operator,

$$\langle\overline{\Psi}\Psi\rangle_{\theta p} = - (N_C/2\pi) \int dp \cos(\theta_p) \qquad [4]$$

Hence, the massless meson state found by 't Hooft[2] must be identified as the chiral Goldstone boson. This result is inconsistent with the usual no-ordering theorems in two dimensions. However, the condensate in Eq.[4] will disappear when loops involving the Goldstone boson are included; these contributions are higher order in $1/N_C$ but infrared divergent in the chiral limit in two dimensions.

At the minimum for θ_p, there are no pair creation terms left in **H**, and the normal ordering produces a single particle energy for the quarks and antiquarks,

$$E_p = p \cdot \sin(\theta_p) + m \cdot \cos(\theta_p) + \alpha_C \cdot N_C \cdot (1/2) \int dq\, [(\cos(\theta_p - \theta_q))/(p-q)^2] \qquad [5]$$

Our solution for the gap equation and the quark self-energy energy can also be obtained from the Schwinger-Dyson equations where the inverse quark propagator is given by $S_F^{-1}(p) = \gamma_0 p_0 - \gamma_1 E_p \sin(\theta_p) - E_p \cos(\theta_p)$. However, it is essential to use the correct $i\varepsilon$ prescription which may require that particles have negative energy at low momentum. This can easily be seen from the expression for the particle energy in Eq.[5]. If we use the principle value definition of the coulomb singularity, then the interaction term can yield a large negative contribution at low energy. The quarks are apparently tachyonic or even have negative energy near the chiral limit. This result would seem absurd as we would to be able to lower the vacuum energy by adding quarks or antiquarks to the vacuum state we have already constructed. Indeed, if the usual cluster properties were to hold, this would be true. However, this theory has confinement because of the linear coulomb potential, and the usual cluster properties are not valid. The attempt to lower the vacuum energy using states with low momentum quarks and antiquarks will result a large, positive interaction energy due to the confining coulomb interactions. This prescription is no mistake as I have established the vacuum stability directly from a variational calculation. These questions can analysed further by studying the elementary excitations which occur above the variational groundstate.

The elementary excitations are color singlet mesons which are made as relativistic quark-antiquark bound states. These states may be studied by diagonalizing the O(1) terms in the Hamiltonian, **H**, of Eq.[3]; note that the vacuum energy of Eq.[3] was $O(N_C)$. For states with a few quark and antiquark pairs added to the leading N_C vacuum state, the O(1) terms in the Hamiltonian are given by the single particle energies of Eq.[5] and certain terms extracted from the normal ordered four body coulomb interaction. I will study these terms using a large N_C bosonization of QCD which is valid to this order in the large N_C expansion. The coulomb interactions can all be expressed in terms of certain color singlet fermion bilinears which are normalized according to our large N_C analysis. They include meson operators,

$$\mathbf{C}(k,q) = \mathbf{B}_q{}^a \cdot \mathbf{A}_{ka}/(N_C)^{1/2}, \quad \mathbf{C}^+(k,q) = \mathbf{A}^{+a}{}_k \cdot \mathbf{B}^+{}_{qa}/(N_C)^{1/2}, \tag{6}$$

and number operators, $\mathbf{NA}(k,q) = \mathbf{A}^{+a}{}_k \cdot \mathbf{A}_{qa}$, $\mathbf{NB}(k,q) = \mathbf{B}^+{}_{ka} \cdot \mathbf{B}_q{}^a$. Acting on states with a few quark and antiquark pairs added to the leading N_C vacuum, the meson operators create and destroy properly normalized color singlet pairs, and the number operators have matrix elements of O(1). In the large N_C limit these meson operators become canonical boson operators with commutation relations,

$$[\mathbf{C}(k,q), \mathbf{C}^+(k',q')] = \delta(k-k')\delta(q-q') + O(1/N_C) \tag{7}$$

In evaluating the normal-ordered coulomb interactions, the O(1) terms are only those which involve the meson operators,

$$\begin{aligned} \mathbf{V} &= (g^2{}_C/2) \int dx \int dy :\mathbf{J}_0{}^a(x) \cdot \mathbf{J}_0{}^a(x): (-\nabla^2)^{-1}(x,y) \\ &= -\alpha_C \cdot N_C \int dp' \int dp \int dq' \int dq \\ &\quad \{ \delta(p'+q'-p-q) \cdot \mathbf{C}^+(p',q')\mathbf{C}(p,q) \cdot [\cos((\theta_{p'}-\theta_p)/2) \cdot \cos((\theta_{q'}-\theta_q)/2)/(p'-p)^2] \\ &\quad + \delta(p'+q'+p+q) \cdot \mathbf{C}^+(p',q')\mathbf{C}^+(q,p) \cdot [\sin((\theta_{p'}+\theta_p)/2) \cdot \sin((\theta_{q'}+\theta_q)/2)/(p'+p)^2] \\ &\quad + \delta(p'+q'+p+q) \cdot \mathbf{C}(p',q')\mathbf{C}(q,p) \cdot [\sin((\theta_{p'}+\theta_p)/2) \cdot \sin((\theta_{q'}+\theta_q)/2)/(p'+p)^2] \} \end{aligned} \tag{8}$$

$+ O(1/N_c)$

The potential terms are quadratic in the canonical boson operators. The kinetic energy remains quadratic in the fermion operators,

$$\mathbf{H}_{0f} = \int dp\, E_p\, \mathbf{A}^{+a}{}_p \cdot \mathbf{A}_{pa} + \int dp\, E_p\, \mathbf{B}^{+}{}_{pa} \cdot \mathbf{B}_p{}^a \qquad [9]$$

I can now use a trick to replace the fermion kinetic energy by a boson kinetic energy which is equivalent to this order in the N_c expansion. The boson kinetic energy,

$$\mathbf{H}_{0b} = \int dp' \int dp\, (E_{p'} + E_p)\, \mathbf{C}^+(p',p)\mathbf{C}(p',p) \qquad [10]$$

has exactly the same matrix elements for color singlet meson states as the fermion operator in Eq.[9].

The full Hamiltonian, $\mathbf{H} = \mathbf{H}_{0b} + \mathbf{V}$, is now a quadratic form in canonical boson operators and may be diagonalized by a boson Bogoliubov transformation of the form,

$$\mathbf{C}(p,q) = \sum_n \{\, \alpha_n(p,q) \cdot \mathbf{D}_n(p+q) + \beta_n(p,q) \cdot \mathbf{D}^+{}_n(-p-q) \,\} \qquad [11]$$

where n labels the meson bound states and $\{\alpha_n(p,q),\ \beta_n(p,q)\}$ are the bound state meson wavefunctions. The confining coulomb potential produces an approximately linear spectrum of meson bound states as found by 't Hooft[2] in the light-cone gauge. The coulomb singularity observed in the single particle energy of Eq.[5] contributes to the boson kinetic energy operator but is precisely cancelled by a similar term in the potential operator of Eq.[8] which comes from the $p' \to p$ limit of the $\mathbf{C}^+(p',q')\mathbf{C}(p,q)$ term. This is expected since there can be no infrared divergent coulomb singularity generated by adding color singlet quark-antiquark pairs to an otherwise stable vacuum. It was only the artificial separation of the kinetic and potential terms which seemed to produce the coulomb singluarity. This separation does not have any meaning in a confining theory where the quarks do not share a cluster property. It is an interesting feature of this calculation that the lowest energy bound state is massless in the chiral limit, $m \to 0$. For a system with a linear, confining interaction potential there must be a positive binding energy for the meson states. Hence, a massless bound state can be achieved only if the kinetic energy has a negative expectation value in the bound state; this is consistent with our observation of the behavior of the single particle quark energy, E_p, near the chiral limit.

The boson Hamiltonian derived above implies a set of bound state equations for the meson wavefunctions. Although the mesons are quark-antiquark bound states, the physical amplitudes must include mesons which go both forward and backward in time. In other words the boson Bogoliubov transformation used to diagonalize the Hamiltonian introduces boson pairs into the meson states as well as into the vacuum. The vacuum boson pairs produce nontrivial four-quark condensates in addition to the two-quark condensates already found by the leading N_C calculation. It is easily shown that the bound state equations greatly simplify in the infinite momentum frame where pair creation is suppressed and the simple bound state Schrödinger equations of 't Hooft are recovered.

I have noted that the solution exhibits spontaneously broken chiral symmetry which is inconsistent in two dimensions. I have calculated the chiral condensate, $\langle\overline{\Psi}\Psi\rangle$, and found the massless Goldstone boson in the bound state spectrum. These features are expected in the large N_C expansion which suppresses the boundstate Goldstone boson fluctuations as their effective couplings should be $O(1/(N_C/2\pi)^{1/2})$. However, the infrared flucuations of the Goldstone field in two dimensions will compensate this suppression for finite N_C, and the two-quark chiral condensates will be expected to vanish. Of course, this means that there must be many quark-antiquark pairs in the vacuum, and our bosonization assumptions are not quite correct. However, I expect the pairs are only those associated with the Goldstone boson bound state, and the calculation of the heavy meson states will be unaffected.

I have used the regular coulomb gauge to study the vacuum structure and the elementary excitations of QCD in two dimensions. I have systematically diagonalized the QCD Hamiltonian using the large N_C expansion. In leading N_C, I have computed the vacuum wavefunctional for QCD and have shown it to be equivalent to a variational calculation of the vacuum structure. In this order there exists a stable ground state with a nontrivial chiral structure. In next order in the N_C expansion I have studied the elementary exicitations of the system which are the color singlet meson bound states. I have demonstrated that a complete bosonization of QCD can be achieved using the large N_C limit. I have also shown how this bosonization leads directly to a dual meson description of QCD. The massless meson state discovered by 't Hooft should be seen as the Goldstone boson of the spontaneous chiral symmetry breaking which appears in the theory although the chiral condensates must disappear when the Goldstone flucuations are included. This calculation complements the usual light-cone gauge calculation where it is difficult to directly determine information on vacuum structure.

QCD in Four Dimensions: Gluons

The perturbative treatment of quantum field theory is usually described in terms of an oscillator, or particle, basis for the quantum fields. Feynman[4] has emphasized the relevant features of the ground state wavefunctional for qualitative questions such as confinement and chiral symmetry breaking. I will look at properties of the vacuum wavefunctional that have an impact on the valence gluon structure.

In pure photodynamics, the Hamiltonian is given by

$$\mathbf{H} = (1/2) \int dx \, \{ \, E^2(x) + \underline{\mathbf{B}}^2(x) \, \} \qquad [12]$$

where the magnetic field is given by $\underline{\mathbf{B}}(x) = \underline{\partial} \times \underline{\mathbf{A}}(x)$ and $\underline{\mathbf{A}}(x)$ and $\underline{\mathbf{E}}(x)$ are canonical variables. The exact vacuum state can be represented in terms of the Schrödinger wavefuncional,

$$\mathbf{\Phi}(\underline{\mathbf{A}}) = \exp \{ - (1/4) \int dx \int dy \, \underline{\mathbf{B}}(x) \cdot \underline{\mathbf{B}}(y) \, \Delta(x-y) \} \qquad [13]$$

where the correlation function is $\Delta(x-y) = 1/2\pi^2(x-y)^2$. This gauge invariant, gaussian wavefuncional describes massless, transverse photons.

In the Schrödinger picture of QCD, the vacuum wavefunctional should preserve the nonabelian gauge invariance and reduce to the perturbative gluon theory at short distance. A simple extension of the photodynamic wavefunctional would yield,

$$\mathbf{\Phi}(\underline{\mathbf{A}}) = \exp \{ - (1/4) \int dx \int dy \, \mathbf{B}^a{}_k(x) \cdot \mathbf{S}^{ab}{}_{kl}(x,y,\mathbf{A}) \cdot \mathbf{B}^b{}_l(y) \, \Delta(x-y) \} \qquad [14]$$

where the nonabelian magnetic field is given by $\underline{\mathbf{B}}^a(x) = \underline{\partial}\times\underline{\mathbf{A}}(x) + igf^{abc}\underline{\mathbf{A}}^b(x)\times\underline{\mathbf{A}}^c(x)$ and $\mathbf{S}^{ab}{}_{kl}(x,y,\mathbf{A})$ is an octet string operator. I reinterpret this wavefunction as a transformation from a supervacuum state to the physical ground state.

$$\mathbf{\Phi}(\underline{\mathbf{A}}) = \exp\{-(1/4)\int dx \int dy\ \mathbf{B}^a{}_k(x)\cdot\mathbf{S}^{ab}{}_{kl}(x,y,\mathbf{A})\cdot\mathbf{B}^b{}_l(y)\ \Delta(x-y)\}\ \mathbf{\Phi}_0 \qquad [15]$$

where $\mathbf{\Phi}_0$ is the supervacuum state with $\underline{E}^a(x)\mathbf{\Phi}_0 = 0$. The exponential represents the transformation which puts octet strings into the vacuum wavefunctional where $\Delta(x-y)$ is the weight for the strings of length $(x-y)^2$. In perturbation theory, $\Delta(x-y)$ is the same as the photon theory with logarithmic corrections and determines the long range correlations in the wavefunctional, $\Delta(x-y) \approx 1/2\pi^2(x-y)^2$

For a confining vacuum, we expect no long range correlations in the wavefunctional, and $\Delta(x-y)$ should be damped. Hence, there are no long strings in the vacuum; the short strings are required to reproduce perturbative QCD at short distance. The infrared components of the electric field are expected to annihilate the vacuum state, $\underline{E}^a(x)_{IR}\mathbf{\Phi}(\underline{\mathbf{A}}) = 0$. In principle, I could use the wavefunctional in Eq.[15] as a variational trial state and determine $\Delta(x-y)$ by a variational principle. There are severe difficulties in calculating matrix elements due to the functional measure of the $\underline{\mathbf{A}}(x)$ integrals as well as gauge problems, etc. Some of these problems could be solved through a lattice formulation of the variational problem.

Instead we will consider a quasi-perturbative approach which assumes the approximate validity of a vacuum wavefunctional with a damped correlation function, ie. $\Delta(x-y) = \exp(-\mu|x-y|)/2\pi^2(x-y)^2$. Then, we compute using effective gluon degrees of freedom, ie. an harmonic oscillator approximation. The ground state is characterized by a set of oscillator frequencies for the gluons, $\omega_k = k^2\cdot\Delta(k)$. The infrared singlularity of the perturbative form of $\Delta(k)$ produces the usual linear dispersion relation, $\omega_k = \sqrt{\underline{k}^2}$. The damped correlation function is not infrared singular and gives a quadratic behaviour at low momenta, $\omega_k = k^2\cdot\Delta(0)$. The precise form of the damping is not essential. This behavior is not that of normal massive gluon which would produce, $\omega_k = \sqrt{\underline{k}^2+m^2}$. Instead, the gluon appears as a nonrelativistic, massive particle but with no mass gap. This behaviour is consistent with a confinement picture where the quadratic behaviour could enhance infrared attraction of gluon exchange interactions instead of the usual screening behavior of massive gluons. In color singlet boundstates, the gluons will be dynamically massive and the positive binding energy of the confining interactions will produce a positive rest mass for the glueball boundstates. This should be contrasted with the quark picture discussed for QCD_2 where the negative quark kinetic energy was seen to produce the massless Goldstone boson state.

While some aspects of this picture of gluons are clearly a property of the coulomb gauge formulation, it may give a useful representation of the confining vacuum structure which affects the nature of valence gluons in boundstates. In principle, we could use the wavefunctions of Eq.[14,15] as a basis of a variational calculation to determine the correlation function, $\Delta(x)$. The long range correlations are expected to be trivial while the short range correlations are those of perturbative QCD. Although the vacuum wavefunctional may be simple, an accurate representation of the gauge field measure is needed to compute matrix elements. This measure is difficult to formulate in the continuum and various lattice formulations of the measure are discussed in other contributions to this workshop.

QCD in Four Dimensions: Quarks

A similar analysis of fermion structure is possible but complicated by the expected nature of long range correlations and the requirement of gauge invariance. We may proceed to represent the fermion vacuum wavefunctional as a transformation from a supervacuum state to the physical ground state. The supervacuum state should contain the correct long range correlations, and a transformation should be used to generate the proper short range correlations of perturbative QCD. Gauge invariance must be preserved in the process.

An approximate supervacuum state can be written as the product of the trivial gluon wavefunction, $\mathbf{\Phi}_{g0} = 1$, and a fermion wavefunction $\mathbf{\Phi}_{f0}$. To satisfy the gauss law constraint of gauge invariance, the fermion wavefunction must be an eigenstate of the local color charge density operator, $\mathbf{J}_0{}^a(x)$,

$$G(x)\mathbf{\Phi}_0 = \{\mathbf{D}\cdot\mathrm{E}(x)+\mathbf{J}_0{}^a(x)\}\mathbf{\Phi}_{g0}\cdot\mathbf{\Phi}_{f0} = \mathbf{\Phi}_{g0}\,\{\mathbf{J}_0{}^a(x)\}\mathbf{\Phi}_{f0} = 0. \qquad [16]$$

The long range correlations dictate the particular solution for $\mathbf{\Phi}_{f0}$. For the case of spontaneously broken chiral symmetry, a solution like that of QCD_2 is required, and the fermion vacuum should correspond to that of an infinitely heavy free quark with $\mathbf{A}(x)\mathbf{\Phi}_{f0} = \mathbf{B}(x)\mathbf{\Phi}_{f0} = 0$ where $\mathbf{A}(x)$ and $\mathbf{B}^{+}(x)$ are, respectively, the upper and lower components of the quark field. For a free massive quark, this state must be transformed by an appropriate Bogoliubov transformaton to the physical vacuum state. For the QCD state, this transformation must be made consistent with gauge invariance but preserving the short distance structure of perturbative QCD. An approximate fermion trial state would be given by

$$\mathbf{\Phi}_f = \exp\{ -\int dx \int dy\ \mathbf{\Psi}^{+a}(x)\mathbf{T}_a{}^b(x,y,\mathbf{A})\boldsymbol{\alpha}\cdot\mathbf{D}\mathbf{\Psi}_b(y)\cdot\Delta_f(x-y) \}\ \mathbf{\Phi}_{f0} \qquad [17]$$

where $\mathbf{T}_a{}^b(x,y,\mathbf{A})$ is a color triplet string operator, and $\Delta_f(x-y)$ is the fermion correlation function. This fermionic transformation obviously generates color triplet strings in the vacuum similar the gluonic transformation which generated the color octet strings in Eq.[15]. At short distance, the effects of the color strings are not expected to be important, and the correlation function must be chosen to produce the correct current quark masses. At long distance, the correlation function, $\Delta_f(x-y)$, will be damped as in the gluonic case and long color triplet strings will be suppressed in the ground state wavefunctional. The incomplete Bogoliubov transformaton has the effect of generating the equivalent of a constituent mass for the quarks. From this view, four dimensional quark structure is quite similar to the picture of chiral symmetry breaking found in QCD_2. As in QCD_2, the long distance chiral structure given in Eq.[17] is incomplete as the direct effects of the Goldstone bosons have been neglected. Hence, a qualitative picture of the valence quark and gluon structure can be obtained from the knowledge of the QCD groundstate wavefunctional without detailed solutions. However, the problem of using these wavefunctionals remains a challenge due to measure problems associated with the gauge field integrations in the continuum theory.

Conclusions.

I have shown how the large N_c expansion can be used to obtain a systematic solution of QCD in two dimensions. The gauge interactions produce a nontrivial chiral structure in the groundstate wavefunctional when analysed in leading order in the N_c expansion. By diagonalizing the Hamiltonian in next order in the N_c expansion, the dynamical equations for the complete set of meson bound states are obtained. The large N_c limit permits a systematic bosonization of the quark theory and produces a dual meson theory of QCD.

A somewhat similar approach was used to analyse the vacuum structure in four dimensional QCD. Here the trial groundstate wavefunctional was given as a transformation from a locally color singlet, supervacuum state. The supervacuum state incorporated the correct long range correlations of the physical vacuum state. The effective "Bogoliubov" transformations generated color octet and color triplet strings in the vacuum. In perturbation theory, the vacuum would include infinitely long strings due to the nature of the correlation functions, $\Delta_g(x)$ and $\Delta_f(x)$. In the QCD ground state, these long strings are expected to be damped, but the short strings are still required to produce the perturbative QCD structure at short distance. Hence, we may, indeed, have good knowledge of the ground state wavefunctionals in this formulation as the wavefunctional are trivial at long distance and perturbative at short distance. Unfortunately, to make real use of this knowledge of the QCD vacuum wavefunctional requires good knowledge of the gauge field integration measure which is presently lacking in the continuum formulation of the theory.

Acknowledgements.

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References.

1. This analysis of QCD in two dimensions was developed in 1977 during a visit to the Max Planck Institute for Physics in Munich and was the subject of numerous seminars but has remained unpublished. Further work on the chiral structure of the theory was done in 1981/82 in collaboration with H. Minakata, also unpublished. It formed the basis of the tranverse lattice formulation of QCD which is contained in the following references: W. Bardeen and R. Pearson, Phys. Rev. D14(1976)547; W. Bardeen, R. Pearson and E. Rabinovici, Phys. Rev. D21(1980)1037.

2. G. 't Hooft, Nucl. Phys. B72(1974)461; Nucl. Phys B75(1984)461.

3. C. Callan, N. Coote and D. Gross, Phys. Rev. D13(1976)1649; M. Einhorn, Phys Rev. D14(1976)3451; M. Einhorn and E. Rabinovici, Nucl. Phys. B128(1977)421.

4. R.P. Feynman, Nucl. Phys. B188(1981)479.

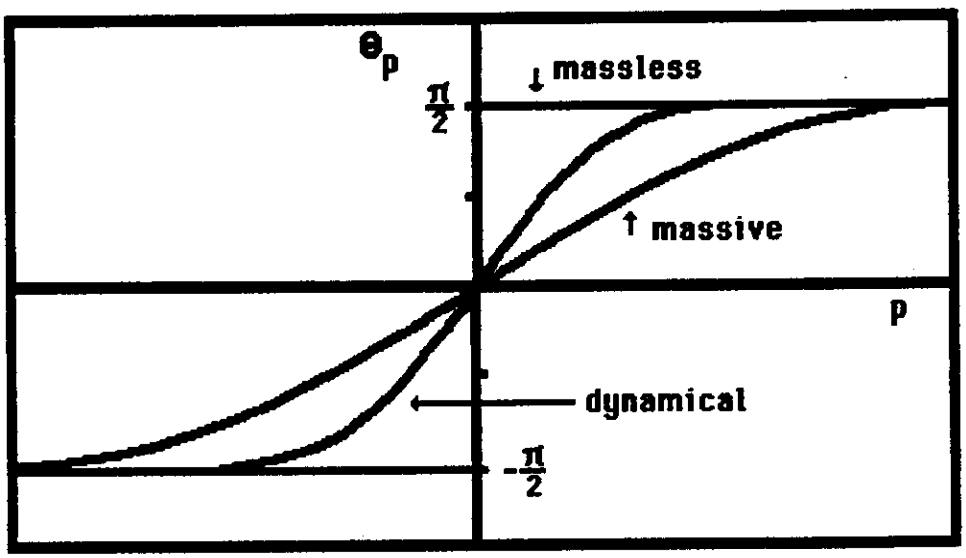

Figure 1. Vacuum chiral angle, θ_p, for massless and massive free fermions and for the dynamically generated chiral angle in massless QCD in two dimensions.